20/12/2014

TDP/BA1–BS1/MTMG/14

TDP (General) 1st Semester Exam., 2014

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer *any* **one** question from each Unit

UNIT—I

1. *(a)* Show that a, b, c are distinct +ve nos.

K-85

$\sum a(b-c)^2 > 0$

$$\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}>6$$ 3

(b) Show that the equation

$$\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$$

represents a rectangular hyperbola $x^2-y^2=xy$. 4

(c) Use Gregory's series to show that

$$\frac{\pi}{8}=\frac{1}{1\cdot 3}+\frac{1}{5\cdot 7}+\frac{1}{9\cdot 11}+\ldots$$ 3

2. *(a)* Find the cube roots of (-1). 3

(b) If a, b and c are positive and not all equal, then prove that

$$\frac{b^2+c^2}{b+c}+\frac{c^2+a^2}{c+a}+\frac{a^2+b^2}{a+b}>a+b+c \quad 3$$

K-101

(c) If $\alpha+i\beta=\cos(\theta+i\phi)$, then prove that

$$\frac{\alpha^2}{\cos^2\theta}-\frac{\beta^2}{\sin^2\theta}=1 \text{ and}$$

$$\frac{\alpha^2}{\cosh^2\phi}+\frac{\beta^2}{\sinh^2\phi}=1 \quad 4$$

UNIT—II

3. *(a)* Prove that for any triangle ABC

$$a^2=b^2+c^2-2bc\cos A$$

where a, b and c denote the lengths of the sides of the triangle. 5

(b) Prove that the points $-2\hat{i}+3\hat{j}+5\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$ and $7\hat{i}-\hat{k}$ are collinear. 5

4. *(a)* With '×' and '·' having usual meaning, for four vectors $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ and $\vec{\delta}$, show that

$$(\vec{\alpha}\times\vec{\beta})\cdot(\vec{\gamma}\times\vec{\delta})=\begin{vmatrix}\vec{\alpha}\cdot\vec{\gamma} & \vec{\alpha}\cdot\vec{\delta}\\ \vec{\beta}\cdot\vec{\gamma} & \vec{\beta}\cdot\vec{\delta}\end{vmatrix} \quad 5$$

(b) Prove that the medians of a triangle are concurrent (using the vector method). 5

UNIT—III

5. (a) If A, B and C be three subsets of a set X, then show that

$$A-(B\cap C)=(A-B)\cup(A-C)$$ 3

(b) Define integral domain. Prove that every field is an integral domain. 1+3=4

(c) Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. 3

6. (a) Let Z be the set of all integers. Define a relation R on Z in the following way :

$$R=\{(a, b)\in Z\times Z : a-b \text{ is divisible by } 7\}$$

Show that R is an equivalence relation. 3

(b) Prove that the set $S=\{0, 1, 2, 3, 4\}$ is a ring with respect to the operations of addition and multiplication modulo 5. 3

(c) Solve the following system of equations by matrix method : 4

$$2x-3y+z=1$$
$$x+2y-3z=4$$
$$4x-y-2z=8$$

UNIT—IV

7. (a) A mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$T(x_1, x_2, x_3) =$

$$(x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3)$$

$(x_1, x_2, x_3) \in \mathbb{R}^3$. Show that T is a linear transformation. 5

(b) If W_1 and W_2 be any two subspaces of a vector space V, then prove that $W_1 \cap W_2$ is also a subspace of V in F. 5

8. (a) Define linear sum of two subspaces. Prove that linear sum of two subspaces of a vector space V over a field F is again a subspace of V over F. 1+4=5

(b) If the vectors $(0, 1, a)$, $(1, a, 1)$ and $(a, 1, 0)$ of the vector space $R^3(R)$ be linearly dependent, then find the value of a. 5

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05/01/2016

TDP (General) 1st Semester Exam., 2015

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that

$$\frac{\pi}{8} = \frac{1}{1\cdot 3} + \frac{1}{5\cdot 7} + \frac{1}{9\cdot 11} + \cdots$$ 3

(b) Prove that

K-83

$$\left\{\frac{1}{2}(n+1)\right\}^n > n!$$ 3

(c) State De Moivre's theorem. Find the roots of $x^7 - 1 = 0$. 3+1=4

2. (a) Show that

$$\frac{\pi}{4}=\left(\frac{2}{3}+\frac{1}{7}\right)-\frac{1}{3}\left(\frac{2}{3^3}+\frac{1}{7^3}\right)+\frac{1}{5}\left(\frac{2}{3^5}+\frac{1}{7^5}\right)-\cdots$$

using Gregory's series. 3

(b) Write the mth power theorem. Using the mth power theorem, find the least value of $x^{-2}+y^{-2}+z^{-2}$, when $x^2+y^2+z^2=9$. 3+1=4

K-116

(c) If $\tan\log(x+iy)=a+ib$, where $a^2+b^2\neq 1$, then prove that

$$\tan\log(x^2+y^2)=\frac{2a}{1-a^2-b^2}$$ 3

UNIT—II

3. (a) Show by vector method that the semi-circular angle is a right angle. 4

(b) If $\vec{p}=(-3, 7, 5)$, $\vec{q}=(-5, 7, -3)$ and $\vec{r}=(7, -5, -3)$, then find $\vec{p}\times(\vec{q}\times\vec{r})$. 3

(c) If the vertices A, B, C of the triangle ABC are defined by position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, then show that the vector area of the triangle ABC is

$$\frac{1}{2}(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})$$ 3

4. (a) Prove that $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$, $\vec{a}$, $\vec{b}$, $\vec{c}$ being three vectors. 3

(b) Find the torque about the point $B(3, -1, 3)$ of a force $P(4, 2, 1)$ passing through the point $A(5, 2, 4)$. 4

(c) If $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$, $\vec{\delta}$ are vectors such that $\vec{\alpha}\times\vec{\beta}=\vec{\gamma}\times\vec{\delta}$ and $\vec{\alpha}\times\vec{\gamma}=\vec{\beta}\times\vec{\delta}$, then show that the vectors $\vec{\alpha}-\vec{\delta}$ and $\vec{\beta}-\vec{\gamma}$ are collinear. 3

UNIT—III

5. (a) Show that the mapping $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x+3$, $x\in\mathbb{R}$ is bijective. Find f^{-1}. 3+1=4

(b) Prove that the set $S=\{1, -1, i, -i\}$ forms a cyclic group under multiplication. 3

(c) What do you mean by a field? Give an example of a ring which is not a field. 3

6. (a) Define equivalence relation. If R be a relation in the set of integer z defined by

$$R=\{(x, y): x+z,\ y\in z,\ (x-y) \text{ is divisible by } 6\}$$

3+1=4

(b) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then verify that A satisfies its own characteristic equation. 3

(c) If R be a ring with unity 1, then prove that this is unique multiplicative identity. 3

UNIT—IV

7. (a) Show that $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T(x, y) = (x + y, x - y), \ (x, y) \in \mathbb{R}^2$$

is a linear transformation. 3

(b) Define kernel of vector space. Show that kernel of a linear transformation $T : V \to W$ is a subspace of V. 1+3=4

(c) Find the eigenvalues of $A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$ and the eigenvector corresponding to the smallest eigenvalue. 3

8. (a) Define a basis of a vector space and show that (1, 0, 1), (0, 1, 1) and (1, 1, 0) form a basis of R^3. 1+4=5

(b) If the linear transformation $T : V \to W$ be such that $\ker(T) = \{0\}$, then prove that the image of a linearly independent set of vectors in V is linearly independent in W. 5

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2 2/12/16

S-1/MTMG/01/16

TDP (General) 1st Semester Exam., 2016

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that the equation

$$\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$$

represents the rectangular hyperbola $x^2-y^2=xy$. 3

(b) Show that

$$8xyz<(1-x)(1-y)(1-z)<\frac{8}{27}$$

when $x+y+z=1$. 3

(c) Show that the product of all values of $(1+i\sqrt{3})^{3/4}$ is 8. 4

2. (a) Show that

$$\pi = 2\sqrt{3}\left(1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\dots\right)$$

using Gregory's series. 3

(b) If $a_1, a_2, \dots, a_n$ be n positive rational numbers whose sum is s, then show that

$$\left(\frac{s}{a_1}-1\right)^{a_1}\left(\frac{s}{a_2}-1\right)^{a_2}\left(\frac{s}{a_3}-1\right)^{a_3}\dots\left(\frac{s}{a_n}-1\right)^{a_n} \le (n-1)^s$$

4

(c) If $x+\frac{1}{x}=2\cos\frac{\pi}{7}$, then show that

$$x^7+\frac{1}{x^7}=-2$$

3

UNIT—II

3. (a) Find by vector method, the area of the triangle having vertices $A(1, 3, 2)$, $B(2, -1, 1)$ and $C(-1, 2, 3)$. 3

(b) Find a vector $\vec{\delta}$ satisfying $\vec{\delta}\cdot\vec{\alpha}=9$ and $\vec{\delta}\times\vec{\beta}=\vec{\gamma}$, where $\vec{\alpha}=2\hat{i}+\hat{j}-\hat{k}$, $\vec{\beta}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{\gamma}=13\hat{i}-10\hat{j}-11\hat{k}$. 3

(c) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors, then prove that

$$[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$$

When are these three vectors in collinear? 3+1

4. (a) Show that the four points whose position vectors are $6\hat{i}-7\hat{j}$, $16\hat{i}-29\hat{j}-4\hat{k}$, $3\hat{j}-6\hat{k}$, $2\hat{i}+5\hat{j}+10\hat{k}$ are coplanar. 3

(b) A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. Find the work done by the forces on the particle. 3

(c) Prove that the medians of a triangle are concurrent (using vector method). 4

UNIT—III

5. (a) Show that the map $f: Q \to Q$ defined by $f(x)=3x+2$ is one-one onto, where Q is the set of rational numbers. 3

(b) If

$$G=\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a \text{ is any non-zero real number}\right\}$$

then show that G forms a commutative group under matrix multiplication. 3

(c) Define integral domain. Show that a set of all integers forms an integral domain but not a field. 4

6. (a) Whether the set $\{1, \omega, \omega^2\}$, where ω is the cube-root of unity, forms a group under the operation multiplication? 3

(b) Find the eigenvalues and the eigenvector corresponding to the largest eigenvalue of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

3

(c) If R be a ring such that $a^2 = a, \forall\ a \in R$, then prove that $a + a = a$ and $ab = ba$, $b \in R$. 4

UNIT—IV

7. (a) Define basis of a vector space. Is the basis of a vector space unique? Justify your answer. 1+4

(b) Define linear transformation. If $T: R^3 \to R$ be defined such that

$$T(a, b, c) = 2a - 3b + 4c$$

then show that T is linear transformation. 1+4

(Continued)

8. (a) Define subspace of vector space. Show that intersection of two subspaces is always a subspace of the same. 1+4

(b) Define range of a linear transformation. Prove that the range of a linear transformation is a subspace. State the rank nullity theorem of a linear transformation. 1+3+1

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21/11/17

TDP (General) 1st Semester Exam., 2017

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* If a and b be positive, then show that

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2 \geq 12\frac{1}{2}$$

where $a+b=1$. 3

(b) If $x=\cos\theta+i\sin\theta$ and $y=\cos\phi+i\sin\phi$, then prove that

$$\frac{x^m}{y^n}+\frac{y^n}{x^m}=2\cos(m\theta-n\phi)$$

where m and n are integers. 3

(c) Expand

$$\tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$$

as a power series in $\tan\theta$. 4

2. (a) If a, b, c be positive unequal, then show that

$$\frac{b^2+c^2}{b+c}+\frac{c^2+a^2}{c+a}+\frac{a^2+b^2}{a+b}\geq a+b+c$$ 3

(b) Find all the values of $(1+i)^{1/3}$. 3

(c) If $z_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, then prove that

$$\lim_{n\to\infty}(z_1, z_2, z_3 \cdots z_n) = -1$$ 4

UNIT—II

3. (a) Prove by vector method that in a triangle ABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

where $BC = a$, $CA = b$ and $AB = c$. 4

(b) Using vector method, show that the line segment joining the mid-points of two sides of a triangle is parallel to the third side. 3

(c) Find the vector equation of the plane passing through the points $2i-3j-k$, $5i-7j+9k$ and $2i-j+3k$. 3

4. (a) Show that every subset of linearly independent set of vectors is linearly independent. 3

(b) Show that

$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{c}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix}$$ 4

(c) A particle, acted on by a constant force (4, 1, −3) is displaced from the point A (1, 2, 3) to the point B (5, 4, 1). Find the work done by the force on the particle. 3

UNIT—III

5. (a) Prove that the intersection of two equivalence relations is also an equivalence relation. 4

(b) If H and K be two subgroups of a group G, then prove that $H \cap K$ is also a subgroup of G. What can be said about $H \cup K$? Justify your answer. 3

(c) Show that the ring M_2 of all 2×2 matrices of the form $\begin{pmatrix} 2a & 0 \\ 0 & 2a \end{pmatrix}$ contains divisors of zero but does not contain the unity, if $a, b \in Z$, the set of integers. 3

6. (a) If I be the set of all integers and the relation R be defined over the set I by aRb and if $(a-b)$ be an odd integer, where $a, b \in I$, then verify whether R is an equivalence relation or not. 3

(b) A commutative ring R without unity is an integral domain, if and only if, for a non-zero element $a \in R$

$$a \cdot b = a \cdot c \Rightarrow b = c : b, c \in R$$ 4

(c) Prove that the set $G = \{1, \omega, \omega^2\}$ where ω is the cube root of unity is a multiplicative cyclic group. 3

UNIT—IV

7. (a) Use Cayley-Hamilton theorem to compute A^{-1}, where

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$$ 4

(b) Show that $S = \{(x, y, z) \in R^3 : x + y + z = 0\}$ is a subspace of R^3. 3

(c) Show that the mapping $T : R^2 \to R^3$ defined by

$$T(x, y) = (x - 2y, 12x + 7y, 5x + 2y)$$

is a linear transformation. 3

8. (a) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{pmatrix}$$ 3

(b) Show that the set

$$S = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$$

forms a basis of $V_3(R)$. 3

(c) Show that the mapping $T : V_2(R) \to V_3(R)$ defined as

$$T(x, y) = (2x + y, x - y, 3y)$$

is a linear transformation from $V_2(R)$ into $V_3(R)$. Find rank and nullity of T. 4

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TDP (General) 1st Semester Exam., 2018

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. (a) If $x+y+z=1$, then show that the least value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is 9 and also show that

$$(1-x)(1-y)(1-z)>8xyz$$ 4

(b) Express $\sinh(x+iy)$ in the form of $A+iB$, where x, y, A and B are real. 3

(c) If $x+\frac{1}{x}=2\cos\alpha$, then show that

$$\frac{x^{2n}-1}{x^{2n}+1}=\pm i\tan(n\alpha)$$ 3

2. (a) Using Gregory's series, prove that

$$\frac{\pi}{2}=\sqrt{3}\left(1-\frac{1}{3.3}+\frac{1}{5.3^2}-\frac{1}{7.3^3}+\cdots\right)$$ 4

(b) If a, b, c be positive and $a+b+c=1$, then show that

$$\left(\frac{1}{a}-1\right)\left(\frac{1}{b}-1\right)\left(\frac{1}{c}-1\right)\geq 8$$ 3

(c) Find the general and principal values of $(-1+i)^i$. 3

UNIT—II

3. (a) Prove by vector method that if the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram. 4

(b) Find the unit vector perpendicular to each of $\vec{a}=4\vec{i}+3\vec{j}-\vec{k}$ and $\vec{b}=2\hat{i}-6\hat{j}-3\hat{k}$. 3

(c) Find the vector equation of the straight line passing through $(2, -3, -1)$ and $(8, -1, 2)$. Also find its Cartesian form. 3

4. (a) If $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of three points A, B, C respectively, then show that

$$\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}$$

is perpendicular to the plane ABC. 3

(b) If the two medians of a triangle are equal, then by vector method, show that the triangle is isosceles. 3

(c) Find the torque about the point $A(1, 2, 3)$ of a force of magnitude 5 units acting through the point $B(3, 4, 5)$ in the direction of the vector $2\hat{i}+3\hat{j}+4\hat{k}$. 4

UNIT—III

5. (a) If A, B are two arbitrary sets and U is universal set, then show that $(A\cap B)'=A'\cup B'$; '$'$' stands for complement. 3

(b) Show that set Z of all integers forms a group under the binary operation $(*)$ defined by $a*b=a+b+1$, $a, b\in Z$. 4

(c) Show that in a field F—

(i) $(-x)^{-1} = -x^{-1}$;

(ii) $(-a)(-b)^{-1} = ab^{-1}$;

where x^{-1} is multiplicative inverse of $x \neq 0$. 3

6. (a) If in a group $(G, \cdot); x^2 = e$ (identity) for every $x \in G$, then prove that G is an Abelian group. 3

(b) If f and g be two mappings from $\mathbb{R}$ to $\mathbb{R}$ given by

$$f(x) = x^2 + 3x + 1 \text{ and } g(x) = 2x - 3$$

then show that

$$(f \circ g)(x) = 4x^2 - 6x + 1$$
$$(g \circ g)(x) = 4x - 9$$

where $(f \circ g)(x) = f(g(x))$. 3

(c) Show that the set

$$M = \left\{ \begin{bmatrix} x & y \\ z & u \end{bmatrix} : x, y, z, u \in Z \right\}$$

Z denotes set of integers, forms a ring with unity. 4

(Continued)

UNIT—IV

7. (a) Let A, B be two matrices such that $AB = 0$, where 0 is the null matrix. Does it imply that $A = 0$ or $B = 0$? Give an example in support of your conclusion. 3

(b) Prove that intersection of two subspaces of a vector space over a field is also a subspace. 4

(c) Show that the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by

$$T(x, y) = (x - y, x + y, y)$$

is a linear transformation $R^2 \to R^3$. 3

8. (a) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$ 4

(b) Let the linear mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by

$$T(1, 2) = (3, -1, 5), \ T(0, 1) = (2, 1, -1)$$

Find $T(x, y)$. 3

(c) Define the basis of a vector space. Is the basis of a vector space unique? Justify your answer. 3

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06/12/19

TDP (General) 1st Semester Exam., 2019

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. (a) If a, b are two positive numbers and $a+b=4$, then prove that
$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2 \geq 12\frac{1}{2}$$ 4

(b) If
$$x+\frac{1}{x}=2\cos\frac{\pi}{7}$$
show that
$$x^7+\frac{1}{x^7}=-2$$ 3

(c) Prove that
$$\tan\left(i\log\frac{a-ib}{a+ib}\right)=\frac{2ab}{a^2-b^2}$$ 3

2. (a) If a, b, c are all positive and $abc = k^3$, then prove that

$$(1+a)(1+b)(1+c) \geq (1+k)^3$$ 3

(b) If a, b, c are positive quantities such that the sum of any two is greater than the third, then show that

$$\frac{1}{a+b-c}+\frac{1}{b+c-a}+\frac{1}{c+a-b} > \frac{9}{a+b+c}$$

unless $a = b = c$. 3

(c) If

$$\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right) \text{ and } \cos\phi = \frac{1}{2}\left(b+\frac{1}{b}\right)$$

then show that $\cos(\theta+\phi)$ is one of the values of $\frac{1}{2}\left(ab+\frac{1}{ab}\right)$. 4

UNIT—II

3. (a) Using vector method, prove that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

for any triangle ABC (symbols having their usual meanings). 4

(b) Find the set of vectors reciprocal to $2i+j-k$, $3i+2j+k$ and $i-j+2k$. 3

(c) Find the vector equation of the plane passing through the points $2\hat{i}-3\hat{j}-\hat{k}$, $5\hat{i}-7\hat{j}+9\hat{k}$, $2\hat{i}-\hat{j}+3\hat{k}$. 3

4. (a) Find the vector $\vec{\alpha}$ which is orthogonal to $\vec{a} = 4\hat{i}-\hat{j}-2\hat{k}$ and $\vec{b} = 5\hat{i}+\hat{j}+\hat{k}$ and $\vec{\alpha}\cdot\vec{c} = 24$ where $\vec{c} = \hat{i}-\hat{j}+\hat{k}$. 4

(b) A particle being acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. Find the total work done by the forces. 3

(c) If $\vec{a}+\vec{b}+\vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, find the angle between the vectors $\vec{a}$ and $\vec{b}$. 3

UNIT—III

5. (a) Prove that the necessary and sufficient condition for a non-empty subset S of a group $(G, *)$ to be a subgroup is

$$a \in S,\ b \in S \Rightarrow a * b^{-1} \in S$$

where b^{-1} is the inverse of b in G. 5

(b) Prove that the set of all real numbers of the form $(a+b\sqrt{2})$, where a and b are rational numbers, is a field under usual addition and multiplication. 5

6. (a) Define cyclic group. Give an example of a cyclic group. Prove that a cyclic group is necessarily Abelian. 1+1+2=4

(b) Show that the set

$$S=\left\{\begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix} : x \in R\right\}$$

is a sub-ring of the matrix ring $(M_2(R), +, .)$ of 2×2 real matrices. 3

(c) Show that the relation 'is perpendicular to' over the set of all straight lines in the plane is symmetric but neither reflexive nor transitive. 3

UNIT—IV

7. (a) Let $S=\{(x, y, z) \in R^3 : 3x-y+z=0\}$. Show that S is a subspace of R^3. Find a basis for S. 3+2=5

(b) Let T be a linear transformation from R^2 into itself that maps $(1, 1)$ to $(-2, 3)$ and $(1, -1)$ to $(4, 5)$. Determine the matrix representing T with respect to the base $\{(1, 0), (0, 1)\}$. 4

(c) State Cayley-Hamilton theorem. 1

8. (a) Prove that Kernel of a linear transformation $T: V \to W$ is a subspace of V. 3

(b) Define Hermitian and skew-Hermitian matrices. Prove that eigenvalues of a skew-Hermitian matrix are either purely imaginary or zero. 2+3=5

(c) For what values of a the vectors $(0, 1, a)$, $(1, a, 1)$ and $(a, 1, 0)$ of the vector space $R^3(R)$ are linearly dependent? 2

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22/03/2022

TDP (General) 1st Semester Exam., 2021
(Held in 2022)

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1**, and **one** question from each Unit

Short-type Questions

Answer *all* questions. Each question carries *2* marks. For True or False statements you need to give a proof if you think it is correct and provide a counterexample otherwise. Justification means to provide a proof when correct or a counterexample when wrong. No marks are to be given for answers without justification.

1. *(a)* Give an example of a relation which is reflexive, symmetric but not transitive.

(b) State True or False and justify—"Every Abelian group is cyclic".

(c) Every integral domain is a field. Justify.

UNIT—I

2. (a) Let a, b, c be positive real numbers. Then prove that
$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}>\frac{3}{2} \text{ unless } a=b=c$$

(b) If $2\cos\theta = x+\frac{1}{x}$ and θ is real, then prove that
$$2\cos n\theta = x^n+\frac{1}{x^n}, \quad n\in\mathbb{Z}$$

(c) If $\tan\log(x+iy)=a+ib$, where $a^2+b^2\neq 1$, then prove that
$$\tan\log(x^2+y^2)=\frac{2a}{1-a^2-b^2}$$
4+3+3=10

3. (a) If a, b, c are positive real numbers, then prove that
$$(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2)\geq 9a^2b^2c^2$$

(b) If $z_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$, then prove that
$$\operatorname*{Lt}_{n\to\infty}(z_1z_2z_3\cdots z_n)=-1$$

(c) Prove that
$$\frac{\pi}{4}=\left(\frac{1}{2}+\frac{1}{3}\right)-\frac{1}{3}\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\frac{1}{5}\left(\frac{1}{2^5}+\frac{1}{3^5}\right)-\cdots$$
3+4+3=10

UNIT—II

4. (a) Show by vector methods, that the line joining the mid-points of the two sides of a triangle is parallel to and half of the third side.

(b) If $\vec{a}\cdot\vec{b}=16$ and $\vec{b}=2\hat{i}+\hat{j}+3\hat{k}$, then show that the vector $\vec{a}$ can be expressed as
$$\frac{16}{7}\hat{i}+\frac{8}{7}\hat{j}+\frac{24}{7}\hat{k}$$
provided $\vec{a}$ is collinear with $\vec{b}$.

(c) Prove that $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\,\vec{b}\,\vec{c}]$, $\vec{a}$, $\vec{b}$, $\vec{c}$ being three vectors.
3+4+3=10

5. (a) If the vectors $\vec{\alpha}$ and $\vec{\gamma}$ are perpendicular to each other, then show that the vectors $\vec{\alpha}\times(\vec{\beta}\times\vec{\gamma})$ and $(\vec{\alpha}\times\vec{\beta})\times\vec{\gamma}$ are perpendicular to each other.

(b) If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three unit vectors such that
$$\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$$
then find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, $\vec{b}$ and $\vec{c}$ being non-parallel.

(c) Find the vector $\vec{\delta}$ which is perpendicular to both $\vec{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$, and $\vec{\delta} \cdot \vec{\gamma} = 21$, where $\vec{\gamma} = 3\hat{i} + \hat{j} - \hat{k}$. 3+3+4=10

UNIT—III

6. (a) Prove that every cyclic group is Abelian.

(b) State and prove the necessary and sufficient condition for a subset of a group to be a subgroup.

(c) A relation f on $\mathbb{R}$ is defined as $(a, b) \in f$ if $a \leq b$. Is the relation reflexive, symmetric and transitive? Justify. 3+(1+3)+3=10

7. (a) Prove that every field is an integral domain.

(b) Show that $c_n = \{\xi : \xi$ is a root of $x^n - 1 = 0,\ n \in \mathbb{N}$ fixed$\}$ is an Abelian group with respect to complex multiplication. Is the group cyclic? Justify.

(c) Can $(\mathbb{Z}_{12}, +_{12})$ have an element of order 7? Justify. 3+(4+1)+2=10

UNIT—IV

8. (a) Show that

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$$

is a subspace of $\mathbb{R}^3$.

(b) Verify whether $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x, y, z) = (x + y,\ y + z,\ z + x)$$

is a linear transformation or not.

(c) State the conditions for which a system of linear equation $AX = B$ has (i) unique solution, (ii) infinitely many solutions, (iii) only zero solution and (iv) no solution. 3+4+3=10

9. (a) Show that any subset of an LI set of vectors in a vector space over a field is linearly independent.

(b) If V and W are vector spaces over a field F, then show that the kernel of an LT, $T : V \to W$ is a subspace of V.

(c) Use Cayley-Hamilton theorem to compute A^{-1}, where $A^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$. 3+3+4=10

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