05/06/2015

TDP (General) 2nd Semester Exam., 2015

MATHEMATICS

(General)

SECOND PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* If

$$\lim_{x\to 0}\frac{\sin 2x + a\sin x}{x^3}$$

be finite, then find the value of a and the limit. 4

(b) Is Rolle's theorem applicable for the function $f(x)=\tan x$ in $[0, \pi]$? Give reasons. 3

(c) Prove that every convergent sequence is bounded. 3

2. *(a)* If $y = \sin(m \sin^{-1} x)$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0 \qquad 4$$

(b) Use Lagrange's mean value theorem to prove that

$$1 + x < e^x < 1 + xe^x \text{ for all } x > 0 \qquad 3$$

(c) Test that the series

$$\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \cdots \quad = \Sigma \frac{(2n-1)}{n(n+1)(n+2)}$$

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is convergent. 3

UNIT—II

3. *(a)* Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$. 3

(b) State Euler's theorem of homogeneous functions of two variables. If

$$u = \frac{x^2y^2}{x+y}$$

apply Euler's theorem to find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} \qquad 4$$

(c) Prove that subtangent at any point of the curve $x^m y^n = a^{m+n}$ varies as abscissa. 3

4. *(a)* Prove that the radius of curvature of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at any point (x, y) is $\frac{(a^4y^2 + b^4x^2)^{3/2}}{a^4b^4}$. 3

(b) Show that the asymptotes of the curve

$$x^2y^2 = a^2(x^2 + y^2)$$

form a square of side $2a$. 4

(c) Find the equation of the normal to the curve $x^3 - 2axy + y^3 = 0$ at the point (a, a). 3

UNIT—III

5. *(a)* Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. 2

(b) If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x \, dx$; m, n being positive integers greater than 1, then prove that

$$I_{m,n} = \frac{n-1}{m+1} I_{m,\, n-2} \qquad 5$$

(c) Using definition of beta function, prove that

$$\int_0^{\pi/2} \cos^4 x\,dx = \frac{3\pi}{16}$$ 3

6. *(a)* Prove that

$$\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^4)}} \times \int_0^1 \frac{dx}{(1+x^4)} = \frac{\pi}{4\sqrt{2}}$$ 4

(b) Prove that the centroid of the whole arc of the cardioid $r = a(1+\cos\theta)$ is

D+M-451
DS→329
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$$\left(\frac{4a}{5}, 0\right)$$ 4

(c) Show that $\Gamma(n+1) = n\Gamma(n)$. 2

UNIT—IV

7. *(a)* Evaluate : 3

$$\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)\,dx\,dy\,dz$$

(b) Find the length of the arc of the curve

$$y = \frac{a}{2}(e^{x/a} + e^{-x/a})$$

between the points $x = 0$, $x = 3$. 4

(c) Determine

$$\iint_R (x^2 + y^2)\,dx\,dy$$

where R is the region bounded by $y = x^2$, $x = 2$, $y = 1$. 3

8. *(a)* Using the transformation $x + y = 4$, $y = 4\theta$, show that

$$\int_0^1 dx \int_0^{1-x} e^{y/x+y}\,dy = \frac{1}{2}(e-1)$$ 5

(b) Find the area of the surface of the solid generated by the revolution of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis. 5

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20/06/2016

TDP (General) 2nd Semester Exam., 2016

MATHEMATICS

(General)

SECOND PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions taking **one** from each Unit

UNIT—I

1. *(a)* State and prove Lagrange's mean value theorem. 1+3=4

(b) If $y=(\sin^{-1}x)^2$, then show that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-x^2y_n=0 \qquad 3$$

(c) Is the series

$$\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\cdots+\left(\frac{n}{2n+1}\right)^n+\cdots$$

convergent or divergent? 3

2. (a) Discuss the convergence of $\sum_{n=1}^{\infty} u_n$, where

$$u_n = \frac{2.4.6.\cdots.(2n+2)}{1.3.5.\cdots.(2n+1)}$$ 3

(b) Expand e^x in powers of x and find the remainder in Lagrange form. 3

(c) Define Cauchy sequence. Prove that every convergent sequence is a Cauchy sequence. 1+3=4

UNIT—II

3. (a) State and prove Euler's theorem for homogeneous functions of two variables. 4

(b) Show that the function $f(x, y) = 4x^2y - y^2 - 8x^4$ has a maximum at (0, 0). 3

(c) Show that the tangent to the curve $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 = 2$ at (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$. 3

4. (a) If $V = \sqrt{X^2 + Y^2 + Z^2}$, show that

$$V_{XX} + V_{YY} + V_{ZZ} = \frac{2}{V}$$ 3

(b) Find the curvature at the point (x, y) on the circle $x^2 + y^2 = a^2$. 3

(c) Find the asymptotes of the curve

$$(x+y)(x-2y)(x-y)^2 + 3xy(x-y) + x^2 + y^2 = 0$$ 4

UNIT—III

5. (a) If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then show that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

and hence deduce the value of I_5. 3+2=5

(b) Show that $\Gamma\left(\frac{7}{2}\right) = \frac{15\sqrt{\pi}}{8}$. 2

(c) Express $\int_0^1 x^m (1-x^n)^p\, dx$ in terms of beta function. 3

6. (a) If $I_n = \int_0^{\pi/2} \sin^n x\, dx$, prove that

$$I_n = \frac{n-1}{n} I_{n-2} \quad (n > 1)$$ 3

(b) Find the length of the arc of the parabola $y^2 = 16x$ measured from $x = 0$ to $x = 4$. 3

(c) Use the relation

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)} \quad (m,\ n > 0)$$

to show that

$$\int_0^1 x^{3/2}(1-x)^{3/2}\,dx = \frac{3\pi}{128}$$ 4

UNIT—IV

7. (a) Evaluate

$$\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}\,dx$$ 4

(b) Evaluate

$$\iint_R \sqrt{x^2+y^2}\,dx\,dy$$

where R is the region bounded by $x^2+y^2=1$ and $x^2+y^2=4$. 4

(c) Evaluate

$$\int_0^{\pi/2}\int_0^{\pi/2} \cos(x+y)\,dx\,dy$$ 2

8. (a) Find the volume and area of the curved surface of the reel generated if the parabola $y^2 = 4ax$ revolves about the tangent at the vertex bounded by the latus rectum.

(b) Evaluate

$$\iint \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}}\,dx\,dy$$

over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ using suitable transformations. 5+5=10

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23/06/17

TDP (General) 2nd Semester Exam., 2017

MATHEMATICS

(General)

SECOND PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions taking **one** from each Unit

UNIT—I

1. *(a)* State and prove Cauchy's mean value theorem.

(b) If $y = e^{a \sin^{-1} x}$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

(c) Test the convergence of the series

$$2 + \frac{3}{8} + \frac{4}{27} + \frac{5}{64} + \cdots + \frac{n+1}{n^3} + \cdots$$ 4+3+3

2. (a) State and prove Leibnitz theorem for successive differentiation of product of two functions of a single variable.

(b) Show that a real sequence is convergent if and only if it is Cauchy.

(c) Using Lagrange's mean value theorem, show that $\log(1+x) > x - \frac{x^2}{2}$ if $x > 0$. 4+3+3

UNIT—II

3. (a) Examine whether $x^{1/x}$ has a maximum or a minimum and if so determine the same.

(b) If the straight line $lx + my = n$ touches the curve $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^p = 1$, then prove that $(al)^{\frac{p}{p-1}} + (bm)^{\frac{p}{p-1}} = n^{\frac{p}{p-1}}$.

(c) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $xu_x + yu_y + \frac{1}{2}\cot u = 0$. 3+4+3

4. (a) Find the radius of curvature at (x, y) on the curve $y = a \log \sec\left(\frac{x}{a}\right)$.

(b) Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ cut orthogonally.

(c) Show that the asymptotes of the curve $(x^2 - y^2)y - 2ay^2 + 5x - 7 = 0$ form an isosceles triangle. 3+3+4

UNIT—III

5. (a) Show that if $I_n = \int \sec^n x\,dx$, then

$$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$$

and hence find I_5.

(b) Assuming $\Gamma(m)\Gamma(1-m) = \pi \operatorname{cosec} m\pi$, $0 < m < 1$, show that

$$\Gamma\left(\frac{1}{9}\right)\Gamma\left(\frac{2}{9}\right)\cdots\Gamma\left(\frac{8}{9}\right) = \frac{16}{3}\pi^4$$

(c) Show that $\Gamma n = \underline{|n-1}$ of $n \in \mathbb{N}$. (3+2)+3+2

6. (a) Find the length of the arc of the curve $x = e^\theta \cdot \sin\theta$, $y = e^\theta \cdot \cos\theta$ measured from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

(b) Show that $\int_0^1 \frac{x\,dx}{\sqrt{1-x^5}} = \frac{1}{5}\beta\left(\frac{2}{5}, \frac{1}{2}\right)$.

(c) Show that

$$\int_0^\infty e^{-x^4} x^2 dx \times \int_0^\infty e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$$ 3+3+4

UNIT—IV

7. (a) Evaluate

$$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz\, dy\, dx$$

(b) Evaluate $\iint xy(x+y)\, dx\, dy$ over the region bounded by $y = x^2$ and $y = x$.

(c) Find the area of the surface generated by revolving about the y-axis the part of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$, that lies in the first quadrant. 3+4+3

8. (a) Find the volume and surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

(b) Using the transformation $x + y = u$, $y = uv$, evaluate the integral $\iint_E e^{y/x+y} dx\, dy$, where E is given by

E : the triangle bounded by $x = 0$, $y = 0$, $x + y = 1$. 5+5

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23/06/18

TDP (General) 2nd Semester Exam., 2018

MATHEMATICS

(General)

SECOND PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer *any* **one** question from each Unit

The symbols used have their usual meanings

UNIT—I

1. *(a)* State and prove Lagrange's mean value theorem.

   (b) If $y = a\cos\log x + b\sin\log x$, then show that

   $$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

   (c) Examine the convergence of the sequence (x_n) defined by

   $$x_n = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots + \frac{1}{\lfloor n}, \text{ for all } n \in \mathbb{N}$$

   (1+3)+3+3=10

2. (a) Show that every convergent sequence is bounded but the converse need not be true in general.

(b) Discuss the convergence of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n}$$

(c) Expand $\log(1+X)$ in powers of X and find the remainder in Lagrange's form.

(3+1)+3+3=10

UNIT—II

3. (a) State and prove Euler's theorem for homogeneous functions of two variables.

(b) Examine for the existence of maxima or minima of the function

$$f(x, y) = x^3 + xy^2 - 12x^2 - 2y^2 + 21x$$

(c) Show that $x\cos\alpha + y\sin\alpha = p$ is a tangent to the curve

$x^m y^n = a^{m+n}$, if

$$p^{m+n} m^m n^n = (m+n)^{m+n} a^{m+n} \sin^n \alpha \cos^m \alpha$$

(1+3)+3+3=10

4. (a) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$$

(b) Find the radius of curvature at the point (r, θ) of the cardioid $r = a(1 - \cos\theta)$ and show that it varies as $\sqrt{r}$.

(c) Find the asymptotes of

$$x^3 + x^2 y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$$

3+3+4=10

UNIT—III

5. (a) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \sin^n x \, dx$ (m and n are positive integers), then show that

$$I_{m,n} = \frac{1}{m+n} + \frac{m}{m+n} I_{m-1,n-1}$$

(b) Show that

$$\int_0^1 \sqrt{1 - x^4}\, dx = \frac{(\overline{|1/4})^2}{6\sqrt{2\pi}}$$

(c) Find the value of $\overline{|9/2}$. 5+3+2=10

6. (a) Find the whole length of the loop of the curve $3ay^2 = x(x-a)^2$.

(b) Evaluate :

$$\int_0^1 x^3 (1 - x^2)^{5/2} dx$$

(c) Define beta function. Using the definition of beta function, prove that

$$\int_0^{\pi/2} \cos^4 x\, dx = \frac{3\pi}{16}$$ 5+3+2=10

UNIT—IV

7. (a) Evaluate $\iint_R \sin(x+y)\, dxdy$ over the region

$$R = \{(x, y) ;\ 0 \le x \le \pi/2,\ 0 \le y \le \pi/2\}$$

(b) Find the volume and surface area of the solid generated by revolving the parabola $y^2 = X$ about the tangent at the vertex bounded by the latus rectum.

5+5=10

8. (a) Find the volume and surface area of the solid generated by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line.

(b) Evaluate

$$\iint \sqrt{\frac{a^2b^2 - b^2x^2 - a^2y^2}{a^2b^2 + b^2x^2 + a^2y^2}}\, dx\, dy$$

over the upper-half of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using suitable substitution.

5+5=10

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02/07/2019

TDP (General) 2nd Semester Exam., 2019

MATHEMATICS

(General)

SECOND PAPER

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer *any* **one** question from each Unit

UNIT—I

1. (a) State and prove Cauchy's mean value theorem.

(b) If $y = \sin(m\sin^{-1} x)$ then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (m^2 - n^2)y_n = 0$$

(c) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)^n}$$

(d) Prove that every convergent sequence is a Cauchy sequence. 3+2+2+3=10

2. (a) Examine the convergence of the sequence (x_n)

$$x_n = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \cdots + \frac{1}{\lfloor n}$$

(b) Expand $\cos x$ in powers of x and obtain the remainder in Lagrange form.

(c) Evaluate :

$$\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x}$$

3+4+3=10

UNIT—II

3. (a) If $V = 2\cos^{-1}\left[\frac{(x+y)}{(\sqrt{x}+\sqrt{y})}\right]$ then show that $xV_x + yV_y + \cot\frac{V}{2} = 0$.

(b) Find all the maxima of $4x^2 - xy + 4y^2 + x^3y + xy^3 - 4$.

(c) Find the double limit and two repeated limits for the function $f(x, y) = \frac{x-y}{x+y}$ at (0, 0), if they exist. 3+3+4=10

4. (a) Find the radius of curvature at θ on the cycloid

$$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$$

(b) Show that the asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$ form a square of side $2a$.

(c) If $lx + my = 1$ touches the curve $(ax)^n + (by)^n = 1$, then show that

$$\left(\frac{l}{a}\right)^{n/n-1} + \left(\frac{m}{b}\right)^{n/n-1} = 1$$

3+3+4=10

UNIT—III

5. (a) Obtain a reduction formula for $\int \tan^n x dx$ and hence show that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

where $I_n = \int_0^{\pi/2} \tan^n x dx$.

(b) Show that

$$\int_0^\infty xe^{-x^8} dx \times \int_0^\infty x^2 e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$$

5+5=10

6. (a) Find the perimeter of the cardioid $r = a(1 - \cos\theta)$ and arc of the upper half of the curve is bisected at $\theta = \frac{2\pi}{3}$.

(b) Define β-function. Show that $\int_0^2 x(8 - x^3)^{1/3} dx = \frac{16\pi}{9\sqrt{3}}$. 5+5=10

UNIT—IV

7. (a) Find the volume and surface area of the solid generated by $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about its base.

(b) Evaluate $\iiint_R xyz\,dx\,dy\,dz$, where R is the positive octant of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

5+5=10

8. (a) Find the volume and surface area of the solid generated by revolving $r = a(1 + \cos\theta)$ about the initial line.

(b) Evaluate $\int_0^1 \left[\int_y^1 e^{x^2}\,dx \right] dy$ by changing the order of integration. 5+5=10

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TDP (General) 2nd Semester Exam., 2022

MATHEMATICS

(General)

SECOND PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Compulsory)

1. Answer the following questions : 2×20=40

(a) Show that the sequence $\left\{\frac{(3n+1)}{(n+2)}\right\}$ is bounded.

(b) In the mean value theorem, $f(a+h)=f(a)+hf'(a+\theta h)$, if $a=3$, $h=5$ and $f(x)=\sqrt{x+1}$, then find θ.

(c) If $u=f(x+ay)+\phi(x-ay)$, then prove that

$$\frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial x^2}$$

(d) Find the value of $\int_0^\infty e^{-x^2} dx$ (assume $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$).

(e) Examine the convergence of the series

$$\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \left(\frac{4}{9}\right)^4 + \cdots$$

(f) Give the statement of Leibnitz's theorem on the *n*th derivative of the product of two functions of *x*.

(g) Define gamma function.

(h) Find the curve whose Cartesian subtangent is constant.

(i) Find the Jacobian of the transformation $x = r\cos\theta$, $y = r\sin\theta$.

(j) Evaluate $\lim_{x \to 0} x \sin\frac{1}{x}$.

(k) State with reason, whether the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

has any asymptote.

(l) Find

$$\lim_{(x, y) \to (0, 0)} \frac{2xy}{x^2 + y^2}$$

if it exists.

(m) State D'Alembert's ratio test for convergence of a series.

(n) Check the continuity of the function

$$f(x) = |x| + |x-1| + |x-2|$$

at $x = 1$.

(o) If $\Gamma(m)\Gamma(1-m) = \pi \operatorname{cosec} m\pi$, $0 < m < 1$, then find the value of

$$\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$$

(p) If $f(x) = \tan x$, is Rolle's theorem applicable to $f(x)$ in $(0, \pi)$? Justify.

(q) Evaluate

$$\iiint xyz \, dx \, dy \, dz$$

over $R[0, 1; 0, 1; 0, 1]$.

(r) State Maclaurin's theorem with Lagrange's form of remainder.

(s) Define Cauchy sequence. Give an example of Cauchy sequence.

(t) State the Young's theorem on function of two variables of partial derivatives.

GROUP—B

Answer **one** question from each Unit

UNIT—I

2. *(a)* State and prove Lagrange's mean value theorem. 5

(b) Test the behaviour of the sequence

$$\left\{\sqrt{5}, \sqrt{5+\sqrt{5}}, \sqrt{5+\sqrt{5+\sqrt{5}}}, \ldots\right\}$$ 5

OR

3. *(a)* If $y = e^{m\sin^{-1}x}$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$$ 3

(b) Prove that every convergent sequence is bounded. Is the converse true? Justify your answer. 3+2

(c) Evaluate $\lim\limits_{x\to 1}\left[\frac{x}{x-1} - \frac{1}{\log x}\right]$. 2

UNIT—II

4. *(a)* State Euler's theorem for homogeneous function of two variables. If u be a homogeneous function of x and y of degree n having continuous partial derivatives, then prove that

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)^2 u = n(n-1)u$$

1½+3½

(b) If ρ_1 and ρ_2 are the radii of curvature at the extremities of any chord of the cardioid $r = a(1+\cos\theta)$ which passes through the pole, then prove that

$$\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$$ 5

OR

5. *(a)* Find the asymptotes of

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$$ 5

(b) Show that the function

$$f(x, y) = 4x^2y - y^2 - 8x^4$$

has a maximum at (0, 0). 3

(c) Find the length of the subtangent of the curve $y = f(x)$ at the point (x, y). 2

UNIT—III

6. (a) If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x\, dx$; m, n being positive integers greater than 1, then prove that

$$I_{m,n} = \frac{n-1}{m+1} I_{m,n-2}$$ 5

(b) Show that

$$\int_a^b (x-a)^m (b-x)^n\, dx = (b-a)^{m+n} \frac{\Gamma(m+1)\,\Gamma(n+1)}{\Gamma(m+n+2)}$$ 5

OR

7. (a) Find the perimeter of the cardioid $r = a(1-\cos\theta)$ and show that the arc of the upper half of the curve is bisected at $\theta = \frac{2\pi}{3}$. 5

(b) Evaluate $\int_0^\infty \frac{dx}{(x+1)(x+2)}$, if it exists. 5

UNIT—IV

8. (a) Evaluate $\iint (x^2 y)\, dx\, dy$ over the positive quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ 5

(b) Find the area of the surface of the solid generated by revolving $x = a(\theta + \sin\theta)$, $y = a(1+\cos\theta)$ about its base. 5

OR

9. (a) Evaluate

$$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$$ 5

(b) Find the centroid of the arc of the parabola $y^2 = 4ax$ included between the vertex and one extremity of the latus rectum. 5

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