02/06/2016

TDP (Honours) 4th Semester Exam., 2016

MATHEMATICS

(Honours)

FOURTH PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. *(a)* Is the following equation exact?

$$(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$$

(b) Solve :

$$x + \frac{p}{\sqrt{1+p^2}} = a$$

where $p = \dfrac{dy}{dx}$.

(c) State the Euler's theorem on homogenous function of x and y of degree n. Verify it on the function

$$f(x, y) = \frac{x^{1/2} + y^{1/2}}{x + y}$$

2+4+4=10

2. (a) Find PI of

$$\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$$

(b) Solve :

$$(D^2 - 1)y = x^2 \cos x$$

(c) Find the equation of the family of curves which cuts the members of the family of hyperbolas $y^2 + 2xy - x^2 = c$ at an angle of 45°.

2+4+4=10

3. (a) Prove that

$$\frac{1}{(x + y + 1)^4}$$

is an integrating factor of

$$(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$$

and hence solve it.

(b) Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$

(c) Solve :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \cdot \sin(\log x)$$

2+4+4=10

UNIT—II

4. (a) Solve :

$$(2x^2 + 3x)\frac{d^2y}{dx^2} + (6x + 3)\frac{dy}{dx} + 2y = (x + 1)e^x$$

(b) Solve

$$x^2\frac{d^2y}{dx^2} - 2(x^2 + x)\frac{dy}{dx} + (x^2 + 2x + 2)y = 0$$

by reducing it to normal form.

5+5=10

5. (a) Solve, by the method of variation of parameters, the equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$$

$y = c_1 x + c_2 x^{-1} + e^x - \frac{e^x}{x}$

(b) Solve

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin x^2$$

by changing the independent variable.

5+5=10

6. (a) Find the eigenvalues and eigenfunctions for the differential equation

$$\frac{d^2y}{dx^2}+\lambda y=0$$

which satisfies the boundary conditions

$$y(0)+y'(0)=0$$
$$y(1)+y'(1)=0$$

(b) Show that the integral of the equations

$$\frac{dx}{dt}=-2y \text{ and } \frac{dy}{dt}=x$$

is given by $x^2+2y+2c=0$. 5+5=10

UNIT—III

7. (a) K-20 Define basic solution of an LPP. Find all the basic solutions of the set of equations given below :

$$2x_1-x_2+3x_3+x_4=6$$
$$4x_1-2x_2-x_3+2x_4=10$$

How many of these BS are BFS?

(b) K-15 SM-46 Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units and 12 units of vitamins A and B per gram respectively and costs 20 paise per gram. The daily requirements of vitamins A and B are at least 100 units and 120 units respectively.

Formulate the above as a linear programming model.

(c) Solve the following LPP using simplex method :

Maximize $Z=4x_1-2x_2-x_3$

subject to

$$x_1+x_2+x_3\le 3$$
$$2x_1+2x_2+x_3\le 4$$
$$x_1-x_2\le 0$$

and $x_1, x_2, x_3\ge 0$ 3+3+4=10

8. (a) Solve by graphical method :

Maximize $Z=4x_1+7x_2$

K-54

subject to

$$2x_1+5x_2\le 40$$
$$x_1+x_2\le 11$$
$$x_2\ge 4$$
$$x_1>0,\ x_2\ge 0$$

(b) =9(a) Prove that the set of all feasible solutions to an LPP $Ax = b,\ x \geq 0$ is a closed convex set.
K-37

(c) What do you mean by 'extreme point' of a convex set? Find all the extreme points of the set
K-26/36

$$S = \{(x, y) \in E^2;\ 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$$

Give an example of a set in E^2 which has *(i)* no extreme points and *(ii)* infinite extreme points. 3+3+4=10

9. K-37 (a) Prove that the set of all feasible solutions of a linear programming problem is a convex set.

(b) Solve the LPP :
K-35

Minimize $Z = 4x_1 + 3x_2$

subject to

$$x_1 + 2x_2 \geq 8$$
$$3x_1 + 2x_2 \geq 12$$
$$x_1 \geq 0,\ x_2 \geq 0$$

by Charnes Big-M method. 5+5=10

UNIT—IV

10. (a) Solve the following LPP by two-phase method :
K-47

Minimize $Z = 4x_1 + x_2$

subject to

$$x_1 + 2x_2 \leq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$3x_1 + x_2 = 3$$
$$x_1,\ x_2 \geq 0$$

(b) Prove that the dual of the dual of a primal LPP is the primal itself. 6+4=10
K-59

11. (a) Solve the following LPP by dual simplex method :
?

Maximize $Z = 3x_1 + 4x_2$

subject to

$$x_1 + x_2 \leq 10$$
$$2x_1 + 3x_2 \leq 18$$
$$x_1 \leq 8$$
$$x_2 \leq 6$$
$$x_1,\ x_2 \geq 0$$

(b) Consider the following problem of assigning four operators to four machines. The assignment costs in ₹ are given below. Find the optimal cost of assignment :

K-117

Operators \ Machines	I	II	III	IV
1	18	26	17	11
2	13	28	14	26
3	38	19	18	15
4	19	26	24	10

5+5=10

12. (a) Find the optimal solution and corresponding cost of transportation in the following transportation problem :

K-99

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

(b) Prove that the following LPP :

K-36

$$\text{Maximize} \quad Z = 5x_1 - 2x_2 + x_3$$

subject to

$$2x_1 + 4x_2 + x_3 \le 6$$
$$2x_1 + x_2 + 3x_3 \ge 2$$
$$x_1,\ x_2 \ge 0$$

and x_3 is unrestricted in sign, admits an unbounded solution. 5+5=10

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03/06/2017

TDP (Honours) 4th Semester Exam., 2017

MATHEMATICS

(Honours)

FOURTH PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. *(a)* State the existence theorem of solution of ordinary differential equation of first order and first degree. 2

(b) Solve : 3

$$xy\left\{\left(\frac{dy}{dx}\right)^2 - 1\right\} = (x^2 - y^2)\frac{dy}{dx}$$

(c) Solve the equation

$$\frac{d^2y}{dx^2} - y\frac{dy}{dx} + 6y = (x-2)e^x$$

by the method of undetermined coefficients. 5

2. *(a)* Determine the family of curves for which the ratio of the y-intercept of the tangent to the radius vector is constant. 5

(b) Solve : 5

$$\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$$

3. *(a)* Solve

$$2x^2 y \frac{d^2y}{dx^2} + 4y^2 = x^2 \left(\frac{dy}{dx}\right)^2 + 2xy \frac{dy}{dx}$$

after making it homogeneous by substitution $y = z^2$. 5

(b) Find the orthogonal trajectories of the family of coaxial circles

$$x^2 + y^2 + 2gx + c = 0$$

where g is the parameter and c is a constant. 5

UNIT—II

4. *(a)* Solve : 5

$$\sin^3 y \frac{d^2y}{dx^2} = \cos y$$

(b) Solve : 5

$$(D+4)x + 3y = t$$
$$(D+5)y + 2x = e^t, \quad D \equiv \frac{d}{dt}$$

5. *(a)* Solve

$$x \frac{d^2y}{dx^2} + (x-2)\frac{dy}{dx} - 2y = x^3$$

by factorization of operators. 5

(b) Solve $(D^2 - 2D)y = e^x \cos x$ by the method of variation of parameters. 5

6. *(a)* Solve

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$$

by reducing it to normal form. 5

(b) Solve : 5

$$\sin^2 x \frac{d^2y}{dx^2} = 2y$$

UNIT—III

7. (a) Solve graphically the following LPP : 5

Minimize $Z = 12x_1 + 20x_2$

subject to

$$6x_1 + 8x_2 \geq 100$$
$$7x_1 + 12x_2 \geq 120$$

and $x_1, x_2 \geq 0$

(b) Prove that a basic feasible solution to a linear programming problem corresponds to an extreme point of the convex set of feasible solutions. 5

8. (a) Solve the following LPP by primal dual method : 5

Maximize $Z = x_1 + 6x_2$

subject to

$$x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

(b) Solve the following LPP by Charnes Big M method : 5

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

subject to

$$2x_1 + 2x_2 - x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_2 + 3x_3 \leq 5$$

and $x_1, x_2, x_3 \geq 0$

(Continued)

9. (a) Solve the LPP :

Maximize $Z = 2x_1 + 4x_2$

subject to

$$x_1 + 2x_2 \leq 5$$
$$x_1 + x_2 \leq 4$$

and $x_1, x_2 \geq 0$

Is this solution unique? If not, write down the convex combination of the alternative optima. 5

(b) Starting from a feasible solution (2, 3, 1) to the set of equations

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$

find out a basic feasible solution. 5

UNIT—IV

10. (a) Solve the following LPP of degeneracy : 5

Minimize $Z = x_1 + x_2$

subject to

$$x_1 + x_2 \geq 5$$
$$3x_1 + x_2 \geq 6$$
$$3x_1 + 2x_2 \geq 6$$

and $x_1, x_2 \geq 0$

(Turn Over)

(b) Use VAM to solve the following transportation problem : 5

	D_1	D_2	D_3	a_i
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
b_j	40	50	60	

11. *(a)* If $\vec{X}^*$ be any feasible solution to the primal LPP. Max $Z = \vec{C}\vec{X}$, subject to $A\vec{X} \le \vec{b}$, $\vec{X} \ge \vec{0}$ and $\vec{V}^*$ be any feasible solution to its dual LPP, then prove that

$$\vec{C}\vec{X}^* \le \vec{b}^T\vec{V}^*$$ 5

(b) Solve the following travelling salesman problem : 5

	A	B	C	D
A	∞	12	10	15
B	16	∞	11	13
C	17	18	∞	10
D	13	11	18	∞

(Continued)

12. *(a)* Obtain the optimal solution and find the corresponding cost of the following transportation problem : 5

	D_1	D_2	D_3	D_4	a_i
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
b_j	6	10	15	4	

(b) Solve the following LPP by two-phase method : 5

Minimize $Z = 4x_1 + 2x_2$

subject to

$$3x_1 + x_2 \ge 27$$
$$x_1 + x_2 \ge 21$$
$$x_1 + 2x_2 \ge 30$$

and $x_1, x_2 \ge 0$

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02/06/2018

TDP (Honours) 4th Semester Exam., 2018

MATHEMATICS

(Honours)

FOURTH PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. *(a)* Find the order and degree of the following differential equation : 2

$$\left\{2+\frac{d^2y}{dx^2}\right\}^{3/4}=3\frac{d^2y}{dx^2}$$

(b) Find the differential equation associated with the primitive $y=a+be^{5x}+ce^{-7x}$, where a, b, c are parameters. 3

(c) Check whether the following equation is exact or not : 2

$$(y^2e^{xy^2}+4x^3)\,dx+(2xye^{xy^2}-3y^2)\,dy=0$$

(d) Solve : 3

$$(x^2y-2xy^2)\,dx-(x^3-3x^2y)\,dy=0$$

2. (a) Reduce $x^2p^2+y(2x+y)p+y^2=0$ to Clairaut's form by the substitution $y=u$, $xy=v$. Hence solve the equation and prove that $y+4x=0$ is a singular solution. 4

(b) Solve : 6

$$(D^2+4)y=x\sin^2 x$$

3. (a) Solve : 5

$$(2x^2+3x)\frac{d^2y}{dx^2}+(6x+3)\frac{dy}{dx}+2y=(x+1)e^x$$

(b) Solve : 5

$$(x+a)^2\frac{d^2y}{dx^2}-4(x+a)\frac{dy}{dx}+6y=x$$

UNIT—II

4. (a) Solve

$$x^2\frac{d^2y}{dx^2}-2(x^2+x)\frac{dy}{dx}+(x^2+2x+2)y=0$$

by reducing it to a normal form. 4

(b) Solve by the method of variation of parameters : 6

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^{2x}$$

(Continued)

5. (a) Knowing that $y=x$ is a solution of the reduced equation of

$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x(1-x^2)$$

solve it after reducing it to a first-order linear equation. 4

(b) Find the eigenvalues and eigenfunctions for the differential equation

$$\frac{d^2y}{dx^2}+\lambda y=0$$

satisfying the boundary conditions $y(0)=0$ and $y(1)=0$. 6

6. (a) Solve : 5

$$(D+5)x+y=e^t$$
$$(D+3)y-x=e^{2t}$$

(b) Solve : 5

$$(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(x+1)e^x$$

UNIT—III

7. (a) A factory is engaged in manufacturing three products—A, B and C, which involve cutting, grinding and assembling. The cutting, grinding and

(Turn Over)

assembling times required for one unit of A are 2, 1 and 1 hour respectively. Similarly, they are 3, 2, 3 hours for one unit of B and 2, 3, 2 hours for one unit of C. The profits on A, B and C are ₹ 50, ₹ 60 and ₹ 90 per unit respectively. Assuming that there are available 300 hours of cutting time, 320 hours of grinding time and 250 hours of assembling time, formulate the problem mathematically so as to find how many units of each product should be produced to maximize the profit. 4

(b) Solve the following LPP by Charnes' Big-M method : 6

Maximize $Z = x_1 + 2x_2$

subject to

$$x_1 - 5x_2 \leq 10$$
$$2x_1 - x_2 \geq 2$$
$$x_1 + x_2 = 10$$
$$x_1,\ x_2 \geq 0$$

8. (a) Prove that the intersection of two convex sets is a convex set. 3

(b) Prove that in E^2, the set $X = \{(x, y) : y^2 \leq x\}$ is a convex set. 2

(c) Find all the basic solutions of the following system of equations : 5

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$

9. (a) Solve the following LPP graphically : 5

Maximize $Z = 4x_1 + 7x_2$

subject to

$$2x_1 + 5x_2 \leq 40$$
$$x_1 + x_2 \leq 11$$
$$x_2 \geq 4$$
$$x_1 > 0,\ x_2 \geq 0$$

(b) Prove that the set of all feasible solutions to an LPP $Ax = b$, $x \geq 0$ is a closed convex set. 5

UNIT—IV

10. (a) Solve the following LPP by the two-phase method : 7

Minimize $Z = 3x_1 + 5x_2$

subject to

$$x_1 + 2x_2 \geq 8$$
$$3x_1 + 2x_2 \geq 12$$
$$5x_1 + 6x_2 \leq 60$$
$$x_1,\ x_2 \geq 0$$

(b) Write down the dual of the following primal and verify that 'dual of the dual is the primal' : 3

Maximize $Z = 2x_1 - 3x_2$

subject to

$$x_1 - 4x_2 \le 10$$
$$-x_1 + x_2 \le 3$$
$$-x_1 - 3x_2 \ge 4$$
$$x_1,\ x_2 \ge 0$$

11. (a) Use VAM to solve the following transportation problem : 5

	D_1	D_2	D_3	D_4	a_i
O_1	6	4	2	7	8
O_2	5	1	4	6	14
O_3	6	5	2	5	9
O_4	4	3	2	1	11
b_j	7	13	12	10	42

(b) Find the optimal assignment for the assignment problem with the following cost matrix : 5

	M_1	M_2	M_3	M_4	M_5
J_1	8	4	2	6	1
J_2	0	9	5	5	4
J_3	3	8	9	2	6
J_4	4	3	1	0	3
J_5	9	5	8	9	5

12. (a) Solve the following travelling salesman problem : 5

	A	B	C	D	E
A	–	12	15	10	8
B	8	–	15	12	8
C	9	11	–	15	11
D	7	12	19	–	11
E	9	12	16	10	–

(b) Solve the following LPP by solving its dual problem by simplex method : 5

Minimize $Z = 3x_1 + x_2$

subject to

$$2x_1 + x_2 \ge 14$$
$$x_1 - x_2 \ge 4$$
$$x_1,\ x_2 \ge 0$$

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03/02/19

TDP (Honours) 4th Semester Exam., 2019

MATHEMATICS

(Honours)

FOURTH PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1\. *(a)* Solve the following differential equation : 3

$$\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3 \cos^2 y$$

(b) Solve : 3

$$x\frac{dy}{dx} + y\log y = xye^x$$

(c) Solve the following differential equation by the method of variation of parameters : 4

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$

2\. (a) Solve the differential equation

$$(D^2 - 2D + 1)y = xe^x \sin x$$

Test the exactness of the following differential equation : 4+2=6

$$(y^2 e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy = 0$$

(b) Find the orthogonal trajectories of the family of coaxial circles

$$x^2 + y^2 + 2gx + c = 0$$

where g is the parameter and c is a constant. 4

3\. (a) Solve by the method of variation of parameter the equation

$$\frac{d^2y}{dx^2} + a^2 y = \sec ax$$ 4

(b) Solve : 4

$$(x\sin x + \cos x)\frac{d^2y}{dx^2} - x\cos x\frac{dy}{dx} + y\cos x = 0$$

(c) Find the particular integral of

$$(D^2 + 1)y = \sin x \sin 2x$$ 2

UNIT—II

4\. (a) Solve the following equation by reducing it to normal form : 5

$$\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(a^2 + \frac{2}{x^2}\right)y = 0$$

(b) Solve : 5

$$(ax + bx^2)\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + 2by = x$$

5\. (a) Find the eigenvalues and eigenfunctions of the differential equation

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad (\lambda > 0)$$

satisfying the boundary condition $y'(0) = 0,\ \ y'(\pi) = 0$. 5

(b) Solve $x\frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} - y = x^2$ by the method of operational factors. 5

6\. (a) Solve the equation

$$x^6\frac{d^2y}{dx^2} + 3x^5\frac{dy}{dx} + a^2 y = \frac{1}{x^2}$$

by changing the independent variable. 5

(b) Solve : 5

$$\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$$

UNIT—III

7. (a) Find a basis of E^3 containing the vectors (1, 1, 2) and (3, 5, 2). 3

(b) Show that although (2, 3, 2) is a feasible solution to the system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$
$$x_1, x_2, x_3 \geq 0$$

It is not a basic solution. How many basic solutions this system may have? 2+1=3

(c) Show that the necessary and sufficient condition for the existence and non-degeneracy of all the basic solutions of $AX = b$ is that every set of m columns of the augmented matrix $[A, b]$ is linearly independent. 4

8. (a) Prove that an LPP has two feasible solutions, then it has an infinite number of feasible solution, as any convex combination of the two feasible solutions is a feasible solution. 4

(b) Solve the following LPP : 4

Minimize $Z = 4x_1 + 8x_2 + 3x_3$

subject to

$$x_1 + x_2 \geq 2$$
$$2x_1 + x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

(c) Give the example of a set having (i) infinite number of extreme points and (ii) no extreme point. 2

9. (a) Solve graphically to show that the LPP

Maximize $Z = 3x_1 + 9x_2$

subject to

$$x_1 + 4x_2 \leq 8$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

admits of a degenerate optimal basic feasible solution. 4

(b) State the fundamental theorem of LPP. 2

(c) Solve the following LPP : 4

Maximize $Z = 2x_1 - 3x_2$

subject to

$$-x_1 + x_2 \geq -2$$
$$5x_1 + 4x_2 \leq 46$$
$$7x_1 + 2x_2 \geq 32$$
$$x_1, x_2 \geq 0$$

UNIT—IV

10. *(a)* Solve the following LPP by two-phase method :

Minimize $Z = 3x_1 + 2x_2$

subject to

$$2x_1 + x_2 \geq 14$$
$$2x_1 + 3x_2 \geq 22$$
$$x_1 + x_2 \geq 1$$
$$x_1,\ x_2 \geq 0$$

Describe a situation when we need not move to phase II, after phase I we will be in a position to conclude the result. 4+1=5

(b) Solve the following assignment problem :

	M_1	M_2	M_3	M_4
J_1	10	24	30	15
J_2	16	22	28	12
J_3	12	20	32	10
J_4	9	26	34	16

Does the problem have any alternative solution? 4+1=5

11. *(a)* State the fundamental duality theorem. 2

(b) By solving the dual of the following problem, show that the following problem has no feasible solution : 4

Minimize $Z = x_1 - x_2$

subject to

$$2x_1 + x_2 \geq 2$$
$$-x_1 - x_2 \geq 1$$
$$x_1,\ x_2 \geq 0$$

(c) Prove that a linear programming problem has a finite optimal solution if and only if there exist feasible solutions to both the prime and dual problems. 4

12. *(a)* Solve the following transportation problem : 5

	D_1	D_2	D_3	a_i
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
b_j	50	80	80	

(b) Find the optimal assignment for a problem with the following cost matrix : 5

	M_1	M_2	M_3	M_4	M_5
J_1	8	4	2	6	1
J_2	0	9	5	5	4
J_3	3	8	9	2	6
J_4	4	3	1	0	3
J_5	9	5	8	9	5

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04/08/2022

TDP (Honours) 4th Semester Exam., 2022

MATHEMATICS

(Honours)

FOURTH PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. (a) Find the integrating factor of the equation
$$(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$$ 2

(b) Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter and c is a constant. 4

(c) Solve the following equation by the method of undermined coefficients : 4
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = x + e^x \sin x$$

(*Turn Over*)

2. (a) Solve by the method of variation of parameters
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$$ 4

(b) Solve :
$$y - x\frac{dy}{dx} = 2(1 + x^2 \frac{dy}{dx})$$
Given $y = 1$ when $x = 1$. 4

(c) Check whether the following equation is exact or not :
$$(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$$ 2

3. (a) Solve :
$$xy\left\{\left(\frac{dy}{dx}\right)^2 - 1\right\} = (x^2 - y^2)\frac{dy}{dx}$$ 3

(b) Solve :
$$x\frac{dy}{dx} + y\log y = xye^x$$ 3

(c) Solve :
$$(x^2D^2 - 3xD + 5)y = x^2 \sin(\log_e x),\ D \equiv \frac{d}{dx}$$ 4

(Continued)

UNIT—II

4. (a) Solve the equation
$$x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$$
in terms of known integral. 5

(b) Solve the simultaneous linear equations
$$\frac{d^2x}{dt^2} + 4x + y = te^{3t}$$
$$\frac{d^2y}{dt^2} + y - 2x = \cos^2 t$$ 5

5. (a) Solve the differential equation
$$\sqrt{x}\cdot\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 3y = x$$
by making it an exact differential equation. 5

(b) Find the eigenvalues and eigenfunctions of the eigenvalue problem
$$\frac{d^2y}{dx^2} - \lambda y = 0,\ (\lambda > 0)$$
with the boundary conditions $y(0) + y'(0) = 0,\ y(1) + y'(1) = 0$. 5

(Turn Over)

6. (a) Solve the equation

$$(y'' + y)\cot x + 2(y' + y\tan x) = \sec x$$

by reduction to normal form. 5

(b) Solve

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (1+x)e^x$$

by the method of operational factors. 5

UNIT—III

7. (a) Prove that $x_1 = 2,\ x_2 = 3,\ x_3 = 0$ is a feasible solution but not a basic feasible of the set of equations

$$3x_1 + 5x_2 - 7x_3 = 21$$
$$6x_1 + 10x_2 + 3x_3 = 42$$

Reduce this feasible solution to a basic feasible solution. 5

(b) Give an example of a convex set. Prove that the objective function of a linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solutions. 1+4=5

8. (a) Find all the basic solutions of the system of equations

$$4x_1 + 2x_2 + 3x_3 - 8x_4 = 6$$
$$3x_1 + 5x_2 + 4x_3 - 6x_4 = 8$$

Also, discuss the nature of each and every basic solution. 4+1=5

(b) Solve the following LPP : 5

Maximize $5x_1 - 2x_2 + 3x_3$

subject to

$$2x_1 + 2x_2 - x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_2 + 3x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

9. (a) Three different types of lorries A, B and C have been used to transport 60 tons solid and 35 tons liquid substance. A type lorry can carry 7 tons solid and 3 tons liquid, B type lorry can carry 6 tons solid and 2 tons liquid, C type lorry can carry 3 tons solid and 4 tons liquid. The costs of transport are ₹ 500, ₹ 400 and ₹ 450 per lorry of A, B and C respectively. To find the minimum cost, formulate the problem mathematically. 4

(b) Prove that in E^2, the set $X = \{(x, y) : y^2 \leq 4x\}$ is a convex set, while the set $X = \{(x, y) : y^2 \geq 4x\}$ is not. 5

(c) State the fundamental theorem of LPP. 1

UNIT—IV

10. (a) If x^* be a feasible solution to the primal problem and v^* be the feasible solution to the dual problem such that $cx^* = b'v^*$, then prove that both x^* and v^* are optimal solutions to the respective problems. 4

(b) State the fundamental duality theorem. 2

(c) Solve the following assignment problem : 4

	I	II	III	IV
1	18	26	17	11
2	13	28	14	26
3	38	19	18	15
4	19	26	24	10

11. (a) Use dual simplex method to solve the following LPP : 5

Maximize $Z = -3x_1 - 2x_2$

subject to

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ x_1 + x_2 &\leq 7 \\ x_1 + 2x_2 &\geq 10 \\ x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(Continued)

(b) Solve the following travelling salesman problem : 5

	A	B	C	D	E
A	–	12	15	10	8
B	8	–	15	12	8
C	9	11	–	15	11
D	7	12	19	–	11
E	9	12	16	10	–

12. (a) Optimize, if necessary, the initial basic feasible solution $x_{11} = 5$, $x_{14} = 2$, $x_{23} = 7$, $x_{24} = 2$, $x_{32} = 8$, $x_{34} = 10$ to the following transportation problem : 5

	W_1	W_2	W_3	W_4	
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
	5	8	7	14	

(Turn Over)

(b) Solve the following LPP by two-phase method : 5

Maximize $Z = 2x_1 + x_2 + x_3$

subject to

$$4x_1 + 6x_2 + 3x_3 \le 8$$
$$3x_1 - 6x_2 - 4x_3 \le 1$$
$$2x_1 + 3x_2 - 5x_3 \ge 4$$
$$x_1, x_2, x_3 \ge 0$$

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